Effects of Variable Fluid Properties and Viscous Dissipation on Mixed Convection Fluid Flow past a Vertical Plate in Porous Medium

P.K.Singh
Department of Mathematics, University of Allahabad, Allahabad-211002, INDIA

Abstract-In the present work, free and forced convection boundary layer flow of an incompressible and viscous dissipative fluid with variable thermal conductivity and temperature dependent viscosity past a an isothermal vertical plate is investigated. The convective flow is taking place in such a porous medium whose permeability is assumed to be spatially variable. The convective flow is due to two factors which influence the flow simultaneously- (1) free stream along the plate and (2) the buoyancy force caused by the variations in density due to temperature difference. The governing equations for the the boundary layer flow are converted into to a system of coupled ordinary differential equations by using suitable similarity transformations. These equations are solved numerically and effects of Grashof number, Eckert number **and** permeability parameter on the velocity and temperature are discussed and presented graphically

Key Words- Porous medium, variable permeability, thermal conductivity, viscosity, viscous dissipation,vertical plate.

1 INTRODUCTION

The combined free and forced convection flow and heat transfer problems in a fluid saturated porous medium have been the subject matter of extensive investigations due to their immense applications in a number of engineering and industrial applications in industry, science and technology- for example in petroleum industry, chemical engineering, geothermal resources and cooling processes of nuclear reactors etc., to name a few. The investigations on free convection flows which were put on firm theoretical foundations by Polhausen[1], were extended by Merkin[2] and [3] and Cheng and Minkowycz [4] etc.. Ranganathann and Viskanta [5] and Chen et al. [6] have studied the combined free and forced convection from vertical plates in porous media.

Here it is mentionworthy that all these studies have been carried out for the fluid flows which have constant properties. The physical properties of the fluids, mainly viscosity and thermal conductivity may change significantly with temperature Schlichting [7]. Kays and Crawferd [8] have described in details various relations between the physical properties of fluids and temperature. It is to be noted that that different fluids behave differently with temperature. Choi [9] studied the effects of variable properties on the boundary layer flow. Lai and Kulaki [10], Pop et al. [11] and Eswara and Nath [12] have shown that temperature dependent viscosity has quite significant effect on momentum and thermal transport in the boundary layer flow. Elbashbeshy [13], Seddeek [14] and Seddeek and Almushigeh [15] considered the hydromagnetic flow and heat transfer past a continuously moving porous boundary with simultaneous effects of radiation and variable viscosity.

Most of the studies involving variable viscosity have considered the thermal conductivity as constants. In order to have a clearer and more insightful picture of the thermal transport, it would be helpful to study the effects of thermal conductivity variation on heat transfer boundary layer phenomena. Elbashbeshy and Ibrahim [16] and Khound and Hazarika [17] have considered the variation of viscosity parameter along with thermal conductivity/ diffusivity parameter and their findings show a significant influence on the velocity and temperature distribution.

Also, the studies of Tierney et al. [18] and Benenati and Brosilow [19] have shown that the porosity of the medium may change which in turn may cause a change in the permeability of the medium. Chandrasekhar et al.[20], took into account the variation of permeability in their investigations on mixed convection flows in the presence of horizontal impermeable surfaces in saturated porous media and have shown that the variability in porosity has significant effect on the velocity distribution and heat transfer. Chandrasekhar and Namboodiri [21] studied the mixed convection flows about inclined surfaces in a saturated porous media incorporating the variation of permeability and thermal conductivity due to packing of particles. Massour and El-Shaer [22] have considered the effects of variable permeability and thermal conductivity while Reddy and Reddy [23] have taken into account the effects of variable viscosity and thermal conductivity on an electrically conducting fluid flow past a moving vertical plate. Hassanien [24] considered the influence of variable permeability and thermal conductivity on the mixed convection flow from an impermeable vertical wedge wherein he obtained a non similarity solution for the case of variable surface heat flux.

In view of the increasing technological applications of mixed convective flows, it would be interesting to study this type of flow with variable fluid properties. To my knowledge, simultaneous effects of variations in viscosity, thermal conductivity and medium permeability have not been studied so far. Thus the aim of the present paper is to investigate the effects of variable permeability and thermal conductivity on the mixed convection boundary layer dissipative flow with temperature dependent viscosity past a vertical plate in porous medium.
a drop cap.

2 MATHEMATICAL FORMULATION

Let us consider a steady two dimensional boundary layer flow of an incompressible and viscous dissipative fluid along a vertical plate in a porous medium . x direction is taken along the plate and y is normal to it ,i.e., the plate starts at x=0 and extends parallel to the x axis and is of semi infinite length. The plate temperature is uniformly maintained at T_w and the temperature T_w is higher than the temperature T_∞ of the fluid far away from the plate. A steady flow parallel to the plate with free stream velocity U_∞ is assumed to take place. The mixed convective flow is assumed to take place due to the simultaneous effects of (1) the free stream along the plate and (2) the buoyancy force caused by the variations in density due to temperature difference. Let u and v be the velocity components in the boundary layer region along the x and y- axes respectively. The viscosity and the thermal conductivity of the fluid are assumed to be variable.Also, the permeability of the porous medium is supposed to be avariable. Then under the usual boundary layer approximations, the governing equations for the present Darcy type flow, following Nield and Bejan [18] and Schlichting [19], are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+g\beta\ T-T_\infty\ +\frac{\mu}{\rho_0 k_0}U_\infty-\frac{\mu}{\rho_0 k(y)}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho_0 c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\mu}{\rho_0 k(y)}u^2 \tag{3}$$

Boundary conditions

$$u=0,\ v=0,\ T=T_w, \quad \text{at} \quad y=0$$
$$u=U_\infty, T=T_\infty, \quad \text{at} \quad y\to\infty \tag{4}$$

We introduce the following non-dimensional variables:

$$\eta=y\left(\frac{U_\infty}{\nu_0\, x}\right)^{\frac{1}{2}}$$

$$\psi=\sqrt{(\nu_0 U_\infty x)}\,f\ \ \eta \text{ , such that } u=\frac{\partial\psi}{\partial y}\ ,\quad v=-\frac{\partial\psi}{\partial x} \tag{5}$$

Now, we have $u=U_\infty f'\ \eta$, $v=\frac{1}{2}\sqrt{\frac{U_\infty\nu_0}{x}}\ \eta f'-f$

And these values of the velocity components satisfy the equa

$$\theta\ \eta\ =\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

The variations in the permeability and thermal conductivity have been assumed, following Chandrasekhara et al.[*], as given below -

$$k(\eta)=k_0(1+d^*e^{-\eta})$$
$$\alpha(\eta)=\alpha_0[\varepsilon_0(1+de^{-\eta})+b\{1-\varepsilon_0(1+de^{-\eta})\}] \tag{7}$$

where d^* and d are constants, α_0 , k_0 and ε_0 are the values of the diffusivity, permeability and porosity respectively at the edge of the boundary layer, b being the ratio of the thermal conductivity of the solid to that of the fluid.

The variation of viscosity with dimensionless temperature, following Slattery (1978), is assumed to be of the form-

$$\mu=\mu_0\,e^{-a\theta} \tag{8}$$

where a is the viscosity variation parameter and it's value depends on the nature of the fluid and μ_0 is viscosity of the fluid at the edge of the boundary layer.

Thus, with these assumptions on the physical parameters given by (7) and (8), the equations (1), (2) and (3) with the help of equations (5) and (6), reduce to the following ordinary differential equations:

$$f'''+\frac{1}{2}e^{a\theta}ff''-a\theta' f''+\frac{Gr}{R_e^{\,2}}e^{a\theta}\theta+\frac{\sigma^2}{R_e}\left(1-\frac{f'}{(1+d^*e^{-\eta})}\right)=0 \tag{9}$$

$$\frac{1}{P_r}\ [\varepsilon_0(1+de^{-\eta})+b\{1-\varepsilon_0(1+de^{-\eta})\}]\theta''+\varepsilon_0 d(b-1)e^{-\eta}\theta'\ +\frac{1}{2}f\theta'+ E_c e^{-a\theta}\left(f''^2+\frac{\sigma^2}{R_e}[\frac{f'^2}{(1+d^*e^{-\eta})}]\right)=0 \tag{10}$$

$$Gr=\frac{g\beta\ T_w-T_\infty\ \ x^3}{\nu_0^{\,2}} \qquad P_r=\frac{\nu_0}{\alpha_0}$$

$$E_c=\frac{U_\infty^{\,2}}{c_p(T_w-T_\infty)} \qquad R_e=\frac{U_\infty x}{\nu_0}$$

$$\sigma^2=\frac{x^2}{k_0} \tag{11}$$

The transformed boundary conditions are given by-

$$\left.\begin{array}{llll} f=0 & f'=0 & \theta=1 & at\ \ \eta=0 \\ f'=1 & \theta=0 & & at\ \ \eta\to\infty \end{array}\right\} \tag{12}$$

The physical problem is now mathematically represented by the equations (6) and (7) and these equations involve five parameters $\frac{Gr}{R_e^{\,2}}, \frac{\sigma^2}{R_e}, P_r$, E_c and δ . Gr being a non dimensional buoyancy parameter gives the free convection while R_e , the Reynolds number gives the forced convection and, therefore, $\frac{Gr}{R_e^{\,2}}$ gives the relative importance of forced and free convection in determining the over all flow. In the similar fashion, the parameter $\frac{\sigma^2}{R_e}$, being a ratio of non dimensional permeability parameter and the Reynolds number represents the relative importance of Darcy and general viscous drag. The Eckert number E_c appears in the viscous dissipation term. The parameter δ appears in the Darcy term.

3 Results and Disscusion

Figure 1(a & b) shows the effects of $\frac{Gr}{R_e^{\,2}}$ on the profiles of the non dimensional velocity f' and temperature θ. As reported in Chandrasekhar et al.[], the flows for the values of $\frac{Gr}{R_e^{\,2}}$ = 0.25 and 0.35, represent the mixed flow with some margin of error. We see that for these values , the velocity (graph a) initially increases from the no slip condition near the plate acquiring a peak value and then gradually decreases and attains the free stream velocity. Same is true for temperature (graph b) also. We have also plotted the boundary layer behaviour for opposing flow also, i.e., for the negative values of $\frac{Gr}{R_e^2}$, here we have taken it as -1. For velocity and temperature both, the profiles steadily approach respectively the values 1 and 0, the far end boundary conditions in the respective cases. While solving the problem, we noticed that the velocity and temperature profiles become oscillatory for the values of $\frac{Gr}{R_e^2} > 0.75$ and to show this kind of behaviour, we have plotted it for $\frac{Gr}{R_e^2}=3$. Also, we have investigated the velocity and temperature distribution taking a=0 which means no variation in the viscosity. In this case, the patterns are like those for $\frac{Gr}{R_e^2}=3$, but with a much greater peak. These unusual behaviours can be attributed to the simultaneous exponential variations of the thermal conductivity, viscosity and medium permeability.

Figure 2 shows the influence of permeability parameter $\frac{\sigma^2}{R_e}$ on the flow field. As the values of this parameter are increased, we observe that velocity and temperature both show increasing trends and then they gradually acquire their respective boundary values. Also, we have plotted these profiles for two values of Pr, 7 & 0, which are shown respectively by dashed and dotted lines.

The effects of Eckert number E_c are shown in the figures 3 and it is observed from both the figures that an increase in E_c has the effect of increasing both velocity and temperature. The effects of Nield modification in the energy equation are shown by the dashed lines and the effect is quite prominent in that the temperature profile has significantly decreased which, in turn, has the effect to lower the velocity profile. The effects of uniform i.e., no variations, in the permeability are shown by the thick lines in both the graphs and, the effects are quite visible in both the profiles. While in the temperature profile, the effect is to increase the temperature, on the other hand, it affects the velocity profile adversely. The effects of constant viscosity are shown by dotted lines and it is observed that velocity field remains almost unaffected but, it has the effects of increasing the temperature profile.

Figure 4 shows the effects of viscosity variations on the velocity and temperature profiles. We know that for the values of a > 0 , the viscosity decreases and from the graph it is noticed that velocity profiles show decreasing patterns corresponding to any increase in the values of a. In both the figures, the effects of clear medium i.e., the absence of prous medium are shown by thick lines showing a decreasing trend in both the profiles. Also, we have investigated the effects of uniform thermal conductivity and medium permeability which are shown by dotted lines and the effects are to lower the velocity and temperature values and this result, perhaps, provides an explanation as to why we got a somewhat unusual pattern with exponential variation of viscosity, thermal conductivity and medium permeability.

4 CONCLUSION

we observe that the velocity initially increases from no slip condition and it acquires a peak value and then gradually decreases and attains the free stream velocity. It is also noticed that for some values of $\frac{Gr}{R_e^2}$, the flow behaviour is somewhat unusual. This may be attributed to the simultaneous variations in the fluid properties. The effects of the parameter $\frac{\sigma^2}{R_e}$, viscosity variation and the permeability variation on the velocity and temperature profiles are clearly visible.

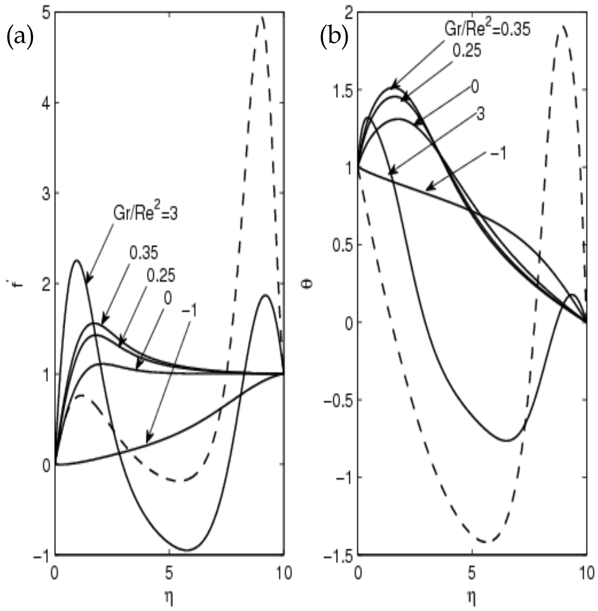

Figure 1

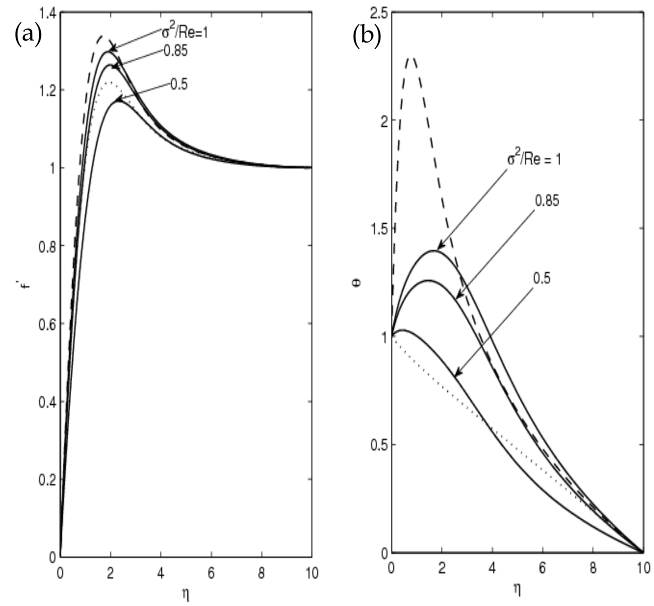

Figure 2

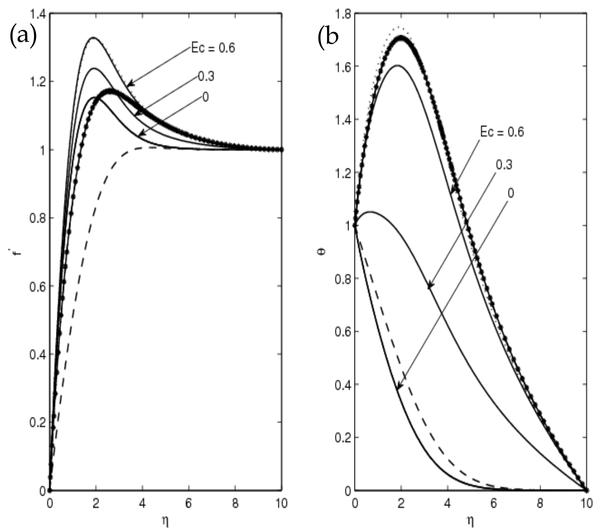

Figure 3

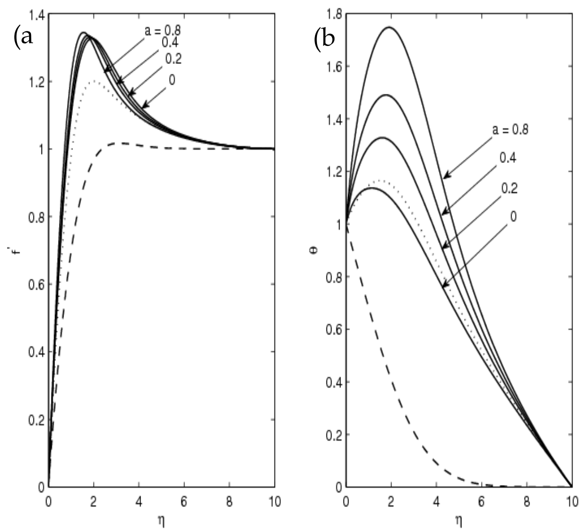

Figure 4

REFERENCES

[1]. K.Pohlhausen, “Zur naherungsweisen Integration der Differentialgleichung der laminaren Grenzschicht”, *ZAMM*, 1921, 252-268

[2]. J.H. Merkin, “The effect of buoyancy forces on the boundary layer flow over a semi infinite vertical flat plate in a uniform free stream”, *J. Fluid Mech*., 35, 1969, 439-450.

[3]. J.H Merkin, “Free convection with blowing and suction”, *Int. J. Heat Mass Transfer*, 15, 1972, 989-999.

[4]. P. Cheng and W.J. Minkowycz, “Free convection about a vertical plate embedded in a porous medium with application to heat transfer from a dick”, *J. Geophys Res*, 28, 1977, 2040-2048.

[5]. P. Ranganathann and R. Viskanta, “Mixed convective boundary layer flow along a vertical surface in a porous medium”, *Num Heat Transfer*, 7, 1984, 305-317.

[6]. C. H. Chen, T. S. Chen and C. K. Chen, “Non-Darcy mixed convection along non isothermal vertical surfaces in porous medium”, *Int. J. Heat Mass Transfer,* 39, 1996, 1157-1164.

[7]. H. Schlichting, “Boundary layer Theory”, *Mc Graw Hill, NewYork*, 1979

[8]. W.M. Kays and M.E. Crawferd, “Convective Heat and Mass Transfer”, *McGraw Hill, NewYork*, 1980.

[9]. I. G. Choi, “The effect of variable properties of air on the

boundary layer for a moving continuous cylinder", *Int. J. Heat Mass Transfer*, 25, 1982, 597-602.

[10]. F. C. Lai and F. A. Kulacki, "The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium", *Int.J. Heat Mass Transfer*, 33, 1990, 1028-1031.

[11]. I. Pop, R. S. R. Gorla and M. Rashid, "The effect of variable viscosity on flow and heat transfer to a continuous moving flat plate", *Int. J. Engg,Sc*., 30, 1992, 1-6.

[12]. A.T. Eswara and G. Nath, "Unsteady non similar two dimensional and axisymmetric water boundary layer with variable viscosity and Prandtl number", *Int. J. Engg,Sc*., 32, 1994, 267-279.

[13]. E. M. A. Elbashbeshy, "Free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presences of the magnetic field", *Int.J.Eng.Sci.* 38, 2000, 207-213.

[14]. M. A. Seddeek, "Effects of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation", *Int. Comm. Heat Mass Transfer*, 27, 2000, 1037-1064

[15]. M. A. Seddeek and A.A. Almushigeh, "Effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction", *Applications and Applied Mathematics*, 5, 2010, 181-197.

[16]. E. M. A. Elbashbeshy and F.N. Ibrahim, "Steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate", *J.Phys*.26 (12), 1993 237-243.

[17]. P. K. Khound and G.C. Hazarika, "The effect of variable viscosity and thermal conductivity on liquid film on an unsteady stretching surface", *Proc. Of 46th Annual Tech. Session, Ass. Sc. Soc*., 2000, 47-56.

[18]. J. W. Tierney, L.H.S. Roblee and R.M. Barid, "Radial porosity variation in packed beds", *A.I.Ch.E. J*., 1958 , 460-464

[19].R.F. Benenati and C.B. Brosilow, "Void fraction distribution in beds of sphere", *A.I.Ch.E. J*., 1962, 359-361.

[20]. B. C. Chandrasekhar, A. R. Hanumanthappa and S. Chandranna, "Mixed convection in the presence of horizontal impermeable surfaces in saturated porous media with variable to permeability", *Proc. 12th Natl Conf Fluid Dynamics Fluid Power*, IIT, Delhi, 1983, Dec.8-10.

[21]. B. C. Chandrasekhar and P. M. S. Namboodiri, "Influence of variable permeability and combined free and forced convection about inclined surfaces in porous media",*Int. J. Heat Mass Transfer*, 28, 1985, 199-206.

[22]. M. Mansour and N. El-Shaer, "Mixed convection radiation in power law fluids along a non-isothermal wedge in a porous medium with variable permeability, Transport in porous Medium" *Transport in porous Medium*, 57 (3), 2004, 336-346.

[23]. M. G. Reddy and N.B. Reddy, "Unsteady MHD convective heat and mass transfer past a ssemi-infinite vertical porous plate with variable viscosity and thermal conductivity", *Inernational Journal of Applied Mathematics and Computation*, 1(2), 2009, 104-117.

[24]. I. A. Hassanien, "Variable permeability effects on mixed convection along a vertical wedge embedded in a porous medium with variable surface heat flux", *Applied Mathematics and Computation*, 138 (1), 41-59, 2003.